Localized Components Analysis


We introduce **Localized Components Analysis** (LoCA) for describing surface shape variation in an ensemble of biological objects using a linear subspace of *spatially localized* shape components.

In contrast to earlier linear subspace methods, LoCA optimizes explicitly for localized components and allows a flexible trade-off between localized and concise representations. For exploratory analyses of ensembles of biological shapes, LoCA allows the presentation of shape variation in an intuitive, anatomically specific way.

Shape characteristics captured by the basis vectors generated with PCA and LoCA:

Top: Arrows start at points tracing a corpus callosum, with their magnitudes indicating the amount that points move when the corresponding shape parameter is varied. The PCA vector shows a global change, while the LoCA vector focuses on the genu.

Bottom: Portions of the ventricles are colored according to the amount that the area changes when the corresponding shape parameter is varied, with darker hues indicating more movement. The PCA vector captures global movement, while the LoCA vector focuses on an occipital horn.

LoCA flexibly trades off the size of the basis against the spatial locality of the basis vectors. The vector capturing the most shape variation in our corpora callosa data is displayed. The concise representation generated by the PCA requires 7 vectors to achieve <10% reconstruction error. Bases with varying degrees of locality exist, with the most localized basis requiring 26 vectors for <10% error.

LoCA optimizes components in terms of compatibility between pairs of surface points.

Coherence between two vertices \( p_i \) and \( p_j \) is defined by the distance from one to the other. Point \( p_i \) is localized around point \( p_c \) if only displacements \( e_{i,j} \) compatible with \( p_c \) are large. Compatibility is expressed by matrix \( B \), which encodes compatibility between points, which can be spatial locality or other higher-order properties, such as spatial symmetry. We select the \( p_c \) that minimizes the cost for each vector.

These costs are computed for each shape component and are summed to give a total cost for each basis. We iteratively optimize the basis by rotating pairs of vectors to minimize a combination of this locality cost and the variance cost minimized by PCA.

Shape components correspond to intuitive modes of shape variation. Examples of fully localized bases generated by LoCA. The first 6 basis vectors are shown (from top to bottom) for corpora callosa, humeri, ventricles, and monkey crania.

To visualize the difference between components A and B, we show models produced by gradually changing the weights from one extreme to the other.

Compatibility can be defined to express higher-order properties, such as spatial symmetry. Points spatially close or symmetric across the midsagittal plane are made highly compatible in matrix \( B \).

Website: [http://idav.ucdavis.edu/~dfalcant/loca.html](http://idav.ucdavis.edu/~dfalcant/loca.html)  
*E-mail: dfalcantara@ucdavis.edu